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DUALITY IN THE ANALYSIS OF RESPONSES TO NONLINEAR SYSTEMS

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Abstract. The paper provides a new view on the averaging procedure and equivalent linearization in the study of nonlinear mechanics. It is shown that the duality of those techniques can be used to obtain better approximate solutions or to separate the original nonlinear systems subjected to periodic and random excitations into deterministic and stochastic ones.

1. INTRODUCTION

There have been extensive investigations in the study of nonlinear dynamic systems due to the fact that all real engineering systems are more or less, nonlinear. Analysis of the response of nonlinear systems subjected to combined periodic and random excitations is considerable interest to the fields of mechanical and structural engineering. The development of the mathematical theory of nonlinear mechanics was started by the classical work of Poincare [1] and followed by many contributions (see for example [2-5]). Among fundamental methods of nonlinear mechanics the averaging method and the method of equivalent linearization have a common technique in order to simplify the problem considered. In fact, these methods are based on the averaging procedure over one period.

In the paper, we provide a new view on the averaging procedure and equivalent linearization. It will be shown that the duality can be used in order to obtained better approximate responses of nonlinear systems. To gain more insight into the new view let us consider Duffing oscillator which drew a great attention in so many investigations.

2. DUALITY IN THE EQUIVALENT LINEARIZATION

The equivalent linearization method (ELM) is a basic tool in the study of nonlinear systems. The idea is to replace in some equivalent sense a nonlinear equation by a linear one since the latter is much easier to analyze. Due to its simple and clear conception the equivalent linearization method is one of the most popular approaches to the approximate analysis.

To describe the basic idea of the equivalent linearization method let consider Duffing oscillator

$$\ddot{z} + 2h\dot{z} + \omega_0^2 z + \gamma z^3 = f(t), \tag{1}$$

where the symbols have their customary meanings $f(t)$ is an external excitation. Following ELM we replace (1) by an equivalent linear equation

$$\ddot{z} + 2h\dot{z} + (\omega_0^2 + \alpha)z = f(t), \tag{2}$$

where the linearization coefficient α is chosen by minimizing mean square equation error

$$< (\gamma z^3 - \alpha z)^2 > \rightarrow \min_{\alpha}, \tag{3}$$

where $< . >$ is the mathematical expectation operator. Condition (3) leads to the expression as follows:

$$\alpha = \gamma \frac{< z^4 >}{< z^2 >} = 3\gamma < z^2 > \tag{4}$$

Eqs. (2), (4) allow to obtain second moments of an approximate response to the original nonlinear (1). Although in general ELM gives a quite good prediction it has been shown, however, by many authors that in the case of major non-linearity the solution error may be unacceptable [5]. In order to reduce the solution error one may use the dual approach to the mean square criterion (3) as follows: go back from kx to $\gamma_1 x^3$ and see how is γ_1 using also the mean square minimum $\gamma_1 z^3$

$$< (\alpha z - \gamma_1 z^3)^2 > \rightarrow \min_{\gamma_1} \tag{5}$$

Eq. (5) yields

$$\gamma_1 = \alpha \frac{< z^4 >}{< z^6 >} \tag{6}$$

or noting (4) one gets

$$\gamma_1 = \gamma \frac{< z^4 >^2}{< z^6 >< z^2 >} = \frac{3}{5}\gamma \tag{7}$$

Hence, using the dual criterion (5) Eq. (1) is replaced by the following linearized equation

$$\ddot{z} + 2h\dot{z} + (\omega_0^2 + \alpha')z = f(t), \tag{8}$$

where α' is found from (4) and (7):

$$\alpha' = \frac{1}{2}(1 + \frac{3}{5})3\gamma < x^2 >= 2.4\gamma < x^2 > \tag{9}$$

Table 1. Approximate second moments of displacements for Duffing oscillator

N	γ	$< z^2 >$	$< z^2 >'$	$< z^2 >_e$
1	0.1	0.805	0.833	0.818
2	1.0	0.434	0.470	0.468
3	10	0.167	0.184	0.189
4	100	0.056	0.063	0.065

To check the accuracy of the equivalent linear equation (8) consider the case where $f(t)$ is a Gaussian while noise excitation for which ($\delta(\tau)$ is the Dirac delta function)

$$< f(t) >= 0, < f(t)f(t+\tau) >= \sigma^2\delta(\tau). \tag{10}$$

Using (10) one gets from Eqs. (2), (4) and Eqs. (8), (9):

$$< z^2 >= \frac{-1+\sqrt{1+12\gamma}}{6\gamma}, < z^2 >' = \frac{-1+\sqrt{1+9.6\gamma}}{4.88},$$

noting that the following values are chosen: $\omega_0 = 1, \sigma^2 = 4h$.

In Table 1, the exact and approximate solutions of Duffing oscillator (1) subjected to the random excitation (10) are listed for different values of γ. It is seen that the solution $< z^2 >'$ is much closer to the exact solutions $< z^2 >_e$ than solution $< z^2 >$ obtained by the conventional equivalent linearization method.

One can unify three procedures (3), (5), (9) into one global criterion as follows

$$< (\gamma z^3 - \alpha z)^2 > + \beta < (\alpha z - \gamma_1 z^3)^2 > + \mu < (\gamma z^3 - \gamma_1 z^3)^2 > \rightarrow \min_{\alpha,\gamma_1}, \tag{11}$$

where β, μ are parameters.

3. DUALITY IN THE AVERAGING PROCEDURE

In this section we consider Duffing oscillator under combined periodic and random excitations

$$\ddot{z} + 2\epsilon h \dot{z} + \omega_0^2 z + \epsilon\gamma z^3 = \epsilon P \sin\nu t + \sqrt{\epsilon} f(t), \tag{12}$$

wherer $f(t)$ is taken as in (10).

Denote

$$z(t) =< z(t) > + u(t). \tag{13}$$

one gets from (13)

$$< u(t) >= 0. \tag{14}$$

Substituting (13) into (12) and taking probabilistic averaging operator to the equation obtained one gers

$$\begin{aligned} &< \ddot{z} > + 2\epsilon h < \dot{z} > + \omega_0^2 < z > + \\ &\qquad + \epsilon\gamma(< z >^3 + 3 < z >< u^2 > + < u^3 >) = \epsilon P \sin\nu t. \end{aligned} \tag{15}$$

Taking again averaging operator with respect to time to the equation and noting (13) - (15) one gets

$$\begin{aligned} &\ddot{u} + 2\epsilon h \dot{u} + \omega_0^2 u + + \epsilon\gamma[3 << z >^2>_t u \\ &\qquad + 3 << z >>_t (u^2 - < u^2 >) + (u^3 - < u^3 >)] = \sqrt{\epsilon} f(t), \end{aligned} \tag{16}$$

where $< . >_t$ denotes the averaging operator with respect to time:

$$< . >_t = \lim_{t\to\infty} \frac{1}{T} \int_0^T (.)dt \tag{17}$$

Hence, by using the dual approach to the averaging procedure the equation (12) can be separated into two equations (15) and (16) for two unknowns $< z >$ and u. It is shown in [4-6] that the equations (15) and (16) can be investigated by the averaging method and equivalent linearization one.

4. CONCLUSION

The main idea of the paper is to show that the dual approach can be used is the analysis of responses to nonlinear systems under periodic and random excitations. It the case of mean square equation error criterion the duality allows to reduce the errror and leads to a better approximate solution. In the case of combined periodic and random excitations the duality allows to separate the original nonlinear equation into two deterministic and stochastic equations, respectively. The advantage of the dual approach will be investigated in more details in next works.

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TÍNH ĐỐI NGẪU TRONG PHÂN TÍCH ĐÁP ỨNG CỦA HỆ PHI TUYẾN

Bài báo trình bày một cách nhìn mới đối với quá trình trung bình hóa và tuyến tính hóa khi nghiên cứu cơ học phi tuyến. Đã chỉ ra rằng sử dụng tính đối ngẫu của các quá trình trên có thể thu được những nghiệm gần đúng tốt hơn hoặc có thể tách hệ phi tuyến chịu kích động tuần hoàn và ngẫu nhiên thành hai hệ tiền định và ngẫu nhiên.